Interaction of an electromagnetic wave packet with an ionization front: Copropagating configuration

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The interaction of a TM polarized wave packet with a moving ionization front is theoretically investigated. We extend our previous study [IEEE Trans. Plasma Sci. **27**, 655 (1999)], where we considered the case when the wave packet is incident on the front, by including the case often used in experiments when the front overtakes the wave packet. We focus on the energy transformation into the generated waves—the point that is rarely addressed in literature due to complications arising from the presence of Langmuir waves. Our quantitative results show the importance of losses via Langmuir wave excitation compared to other possible losses due to the excitation of stationary transverse electron currents in the plasma. Applicability to the generation of frequency upshifted radiation is discussed.

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An electromagnetic wave packet propagating in a stationary gas with the index of refraction $n \approx 1$ can be overtaken by an ionization front if the velocity of the front V satisfies $\beta = V/c > -\cos\theta_0$ where c is the *in vacuo* speed of light and θ_0 is the angle of incidence. The case $\theta_0 > 90°$ is a special case of incidence of a wave packet onto an ionization front and it is typically referred to as the copropagating configuration, in contrast to the case $\theta_0 < 90°$ being the counterpropagating configuration. The interaction of the wave packet with the ionization front results in the formation of a reflected wave packet that propagates in the gas away from the moving front, a transmitted wave packet that is created behind the ionization front, and so-called free-streaming mode that consists of stationary transverse electron currents and stationary magnetic field in the plasma [1–3]. If the electric field of the original wave packet has a component perpendicular to the front (TM, or *p*-polarized wave packet), two Langmuir waves are excited behind the front while no Langmuir-wave excitation occurs for TE (or *s*-) polarization [2,4]. The ionization front carries no energy, so the energy of the original wave packet is split between the generated wave packets. Both the reflected and the transmitted wave packets are frequency upshifted and compressed in duration. This makes the problem of the interaction of electromagnetic wave packets with moving ionization fronts attractive, since it allows straightforward control over the degree of frequency upshift by varying the density of the created plasma or the angle of incidence and thus, creation of tunable sources of coherent radiation can be envisioned [5,6]. In practice, the density of the plasma created by laser pulses is relatively low, that is, the plasma is underdense for the incident radiation. As a result, the reflected wave packet in the counterpropagating configuration, even though being highly frequency upshifted, is of little practical use since it carries only a very small fraction of the incident energy. In the copropagating configuration, the reflected energy can be very large though the frequency upshift becomes small compared to the counterpropagating case. In 1991, Mori [7] pointed out that the transmitted wave can also be highly frequency upshifted. Moreover, for low plasma densities, the maximum frequency upshift of the transmitted wave occurs in the copropagating configuration [5].

In experiments, oblique incidence of a wave packet onto an ionization front can be provided by confining the wave packet in a waveguide as in experiments by Savage *et al.* [8,5]. These experiments reported frequency upshifts up to a factor of 5 of 35 GHz pulses (TE mode of the waveguide) overtaken by the ionization front created by a laser pulse. Recently, the idea of frequency-upshifting was tested for optical frequencies by colliding two laser beams: one produces a moving ionization front, the other, probe beam, is being frequency upshifted due to the interaction with the front [9]. The two configurations, counterpropagating and copropagating, were tested and it was experimentally verified that the frequency upshift is indeed due to the moving ionization front. The polarization of the probe beam was not specified in those experiments.

Even though the degree of frequency upshift is polarization independent, the energy transferred into the frequency upshifted radiation strongly depends on the polarization of the original wave packet. In Ref. [4], we showed that for TM polarized wave packets strong resonant excitation of Langmuir wave occurs at small angles of incidence and at plasma densities when the transmitted wave is expected to reverse its propagation direction in the plasma; the transmitted wave packet completely disappears at exact resonance. The resonant excitation occurs only in a narrow interval of plasma densities. Outside this region, the losses are due to the excitation of the free streaming mode.

In this Brief Report, we extend the study of the interaction of TM polarized wave packets with an ionization front by including the copropagating configuration and giving a quantitative comparison of the energy balance in the two cases—copropagating and counterpropagating configurations. In the copropagating configuration the excitation of the free streaming mode becomes less significant than in the counterpropa-

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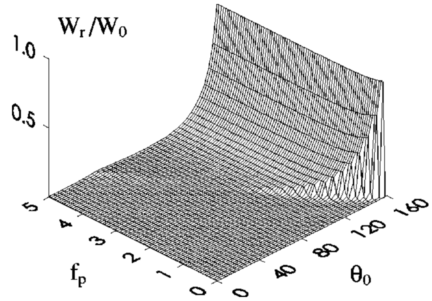

FIG. 1. Energy of the reflected wave packet normalized to the energy of the incident wave packet W_r/W_0 as a function of normalized plasma density $f_p=\omega_p/\omega_0$ and incident angle θ_0 for $\beta=0.941$.

gating configuration, while the excitation of Langmuir waves becomes the dominant mechanism of losses. The excitation of Langmuir waves loses its resonant character and occurs for any plasma density except when the density is very small. The region of small plasma densities is, however, of little interest since the frequency upshift of the transmitted wave is insignificant for any practical applications.

Throughout this paper, the formulas derived in our previous study of the case $\theta_0<90°$ are used [4]. All formulas remain valid for $\theta_0>90°$. For all the results we present below, we verified that the total energy of the excited wave packets is equal to the energy of the incident wave packet. This ensures the credibility of our results.

Let us consider a TM polarized wave packet with carrier frequency ω_0 freely propagating in a gas under an angle θ_0 with respect to the positive x-direction. The wave packet then interacts with an ionization front that moves with velocity V in the negative x direction and creates a plasma with plasma frequency $\omega_p=\sqrt{4\pi Ne^2/m}$, where N is the plasma density, e and m are the electron charge and mass, respectively. We assume that the rise time of the plasma density is infinitely small, that in practice means that it is much smaller than either the period of the incident wave or any of the waves that are generated afterwards. After the process of interaction of the original wave packet with the front, five new wave packets are created: a reflected wave packet, a transmitted into the plasma wave packet, two Langmuir

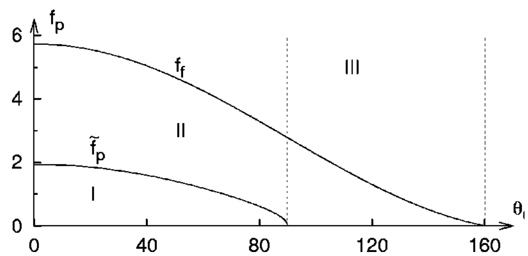

FIG. 2. Regimes of propagation for the transmitted wave packet in the plasma as a function of f_p and θ_0 for $\beta=0.941$. In region I ($f_p<\tilde{f}_p$ where $\tilde{f}_p=\sqrt{(\cos\theta_0+2\beta+\beta^2\cos\theta_0)\cos\theta_0}$), the transmitted wave packet propagates in the positive x direction; in region II ($\tilde{f}_p<f_p<f_f$), the transmitted wave packet runs after the front; in region III ($f_p>f_f$), the transmitted wave exponentially decays in the plasma.

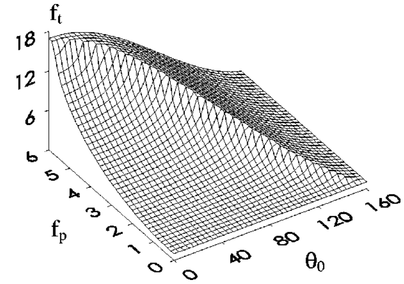

FIG. 3. Normalized frequency $f_t=\omega_t/\omega_0$ of the transmitted wave as a function of f_p and θ_0 for $\beta=0.941$.

wave packets, and a free-streaming wave packet. The frequencies and the angles of propagation of the excited wave packets are found using the condition of continuity of the wave phase across the moving boundary and dispersion equations for the waves in free space and in the plasma. The energies are found from the reflection and transmission coefficients for the cw fields and then taking into account how the wave packets change in shape due to the interaction with the moving front [4].

The reflected wave is double Doppler upshifted with frequency $\omega_r/\omega_0=(1+2\beta\cos\theta_0+\beta^2)/(1-\beta^2)$ and it propagates at the angle θ_r with the negative x direction given by the equation $\sin\theta_r=(\omega_0/\omega_r)\sin\theta_0$. The frequency upshift of the reflected wave is at maximum in the counterpropagating configuration ($\theta_0=0$) and decreases with θ_0 completely vanishing when $\cos\theta_0=-\beta$. The frequency of the reflected wave is independent of the density of the created plasma. The energy of the reflected wave packet is shown in Fig. 1. For this figure as well as for all subsequent figures we have chosen $\beta=0.941$ as a typical value achieved in experiments [9]. At this value of β, the maximum angle at which front can still overtake the wave packet is $\theta_0^{\max}=\cos^{-1}(-\beta)=160.2°$. As we see from Fig. 1, the reflected energy is large only in the region of large angles when, as we will show later, skinning of the transmitted wave in the plasma occurs. The frequency upshift of the reflected wave in this case is small.

The transmitted wave packet in the copropagating configuration always runs after the front or completely disappears if the plasma becomes overdense, i.e., $f_p>f_f$ where $f_p=\omega_p/\omega_0$ and $f_f=(\beta+\cos\theta_0)/\sqrt{1-\beta^2}$. This is in contrast to the counterpropagating geometry when the transmitted wave packet can run both after and from the front (Fig. 2). The frequency of the transmitted wave grows with the plasma density and the angle of incidence and reaches a maximum when the wave starts to decay in the plasma at $f_p=f_f$ (see Fig. 3). If we increase the plasma density above the critical density $f_p>f_f$, the real part of the frequency f_t does not change with the plasma density while the imaginary part increases that corresponds to a stronger skinning of the wave behind the front. In the region of skinning, the real part of f_t decreases with the incident angle θ_0 for a fixed value of the plasma density f_p. Apparently, the energy transmitted into the plasma is zero when the skinning occurs as shown in Fig. 4. Indeed, if we consider the final result of the interaction—when the incident wave packet is gone and the

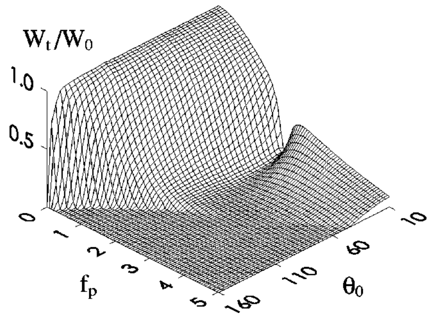

FIG. 4. Energy of the transmitted wave packet W_t/W_0 as a function of f_p and θ_0 for $\beta=0.941$. Note that θ_0 starts at 10°. For smaller angles, the resonant dip shrinks (see Fig. 6 in Ref. [4]).

excited wave packets came off the boundary—there is no transmitted wave packet. The well-defined dip in Fig. 4 at small angles is due to the resonant excitation of Langmuir waves and it is explained in Ref. [4].

Two Langmuir waves excited behind the front have frequencies $\pm\omega_p$ and propagate at the angles $\tan\theta_{l\pm}=\beta\sin\theta_0/(1+\beta\cos\theta_0\mp f_p)$. One of the waves always runs after the front, the other runs after the front if $f_p>1+\beta\cos\theta_0$ and propagates in the positive x direction in the opposite case. The total energy of the Langmuir wave packets is shown in Fig. 5. Clearly, the losses due to the excitation of Langmuir waves are significant. For small angles the excitation of Langmuir waves has a resonant character and occurs only near $f_p\approx 1+\beta$, for large angles the excitation occurs practically for any value of plasma density except very small densities. As the incident angle approaches the critical angle $\theta_0^{\max}\approx 160.2°$, the excitation of Langmuir waves disappears and the incident energy is reflected completely.

Besides the Langmuir wave excitation, there is one more source of losses—excitation of the free-streaming mode. The energy of the free streaming mode is shown in Fig. 6. While the excitation of Langmuir waves plays a significant role for the copropagating case, the excitation of the free-streaming mode decreases with an increase of the incident angle and for the copropagating case is not very important as we see from Fig. 6.

To comprehend the general picture of energy distribution, it is instructive to consider in particular the limit of high plasma density ($f_p\to\infty$). In this limit the energies are given by simple formulas:

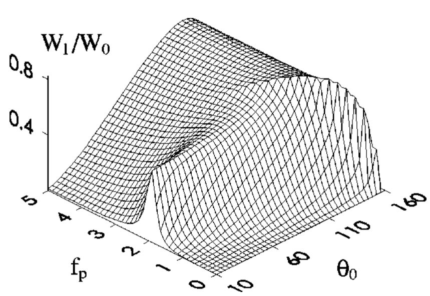

FIG. 5. Energy of the Langmuir wave packets W_l/W_0 as a function of f_p and θ_0 for $\beta=0.941$. Note that θ_0 starts at 10°. For smaller angles, the resonant peak shrinks (see Fig. 6 in Ref. [4]).

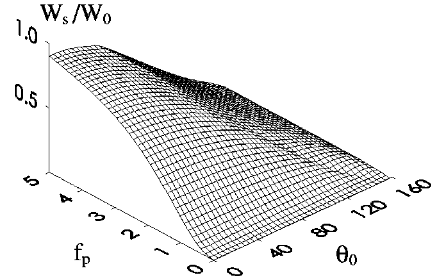

FIG. 6. Energy of the free-streaming wave packet W_s/W_0 as a function of f_p and θ_0 for $\beta=0.941$.

$$W_t=0, \tag{1}$$

$$W_r=W_0\frac{(1-\beta^2)\cos^2\theta_0(1+\beta^2+2\beta\cos\theta_0)}{(2\beta+\cos\theta_0+\beta^2\cos\theta_0)^2}, \tag{2}$$

$$W_s=W_0\frac{2\beta(\beta+\cos\theta_0)(1+\beta\cos\theta_0)^2}{(2\beta+\cos\theta_0+\beta^2\cos\theta_0)^2}, \tag{3}$$

$$W_l=W_0\frac{2\beta\sin^2\theta_0(\beta+\cos\theta_0)}{(2\beta+\cos\theta_0+\beta^2\cos\theta_0)^2}, \tag{4}$$

where W_0 is the incident wave energy; both Langmuir waves carry away the same amount of energy. It is easy to check that $W_t+W_r+W_s+W_l=W_0$ as it is required by the energy conservation.

In application to experiments [5,9], where the copropagating configuration was used, the most interesting region of parameters θ_0 and f_p lies near the limiting angle $\theta_0^{\max}$ at small values of f_p (Fig. 7). In experiments [9], for example, $f_p=0.08$–0.14, $\theta_0\approx 160°$. As a deviation of the incident angle in a few degrees from 160° is unavoidable in a real experiment, it is evident from Fig. 7 that energy losses due to Langmuir waves generation may become significant (up to 40% at $\theta_0\approx 155°$).

To conclude, we have investigated the energy conversion into different modes for the case when an electromagnetic wave packet is overtaken by a subluminous ionization front with a sharp rise time. The approximation of a sharp rise time needs some justification. For ionization fronts created by laser pulses, such as used in experiments reported in Refs. [5,9], the rise time is typically determined by the width of the ionizing pulse that can be as short as 10–100 fs. Manifestly, if the upshifted signal has frequency also in the optical range,

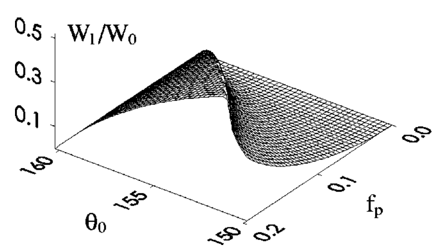

FIG. 7. Same as Fig. 5 for small densities and large incident angles.

the approximation of a sharp front does not hold while it should be valid for incident pulses in the microwave range of spectrum as in Refs. [5,8]. However, recently the ionization of atoms using a few periods of lights (sub-10-fs pulses) has attracted a considerable attention [10]. Such pulses can create ionization fronts with a several femtosecond rise time and the approximation of the instant rise time of the front can be extended to much higher frequencies of the upshifted signal (near infrared and even in the optical range). Certainly, the kinematic properties of the generated wave cannot be affected by a particular shape of the ionization front, while the energy transformation can be different. We have considered the case of TM polarization of the incident wave packet, and we have shown that energy loss into Langmuir waves can be very large. For example, for $\beta = 0.941$, 80% of the incident energy can be lost via Langmuir-wave excitation. Moreover, for the copropagating configuration, the losses are almost independent of the plasma density if the density is relatively large. This is in contrast with the case when a wave packet is incident under small angles onto the front and the Langmuir-wave excitation occurs only in the very narrow interval of plasma densities $\delta\omega_p/\omega_0 \approx \theta_0$ near $\omega_p/\omega_0 \approx 1+\beta$. This difference in the excitation of Langmuir waves for small and large incidence angles may be especially significant for incident wave packets of finite frequency spectrum width. Indeed, parameter f_p has different values for different spectral components. If the packet spectrum is wide enough, the total energy transferred to Langmuir waves may be small for any plasma density in the region of small incident angles. For large incident angles, energy transfer to Langmuir waves will be significant even for short incident pulses with wide frequency spectrum. We also expect that broadening of ionization front may reduce the amount of energy transferred to the Langmuir waves in favor of the free streaming mode. In general, Langmuir waves can be excited either at a sharp plasma boundary or at the point of plasma resonance for smooth fronts. However, the condition of plasma resonance $f_p = (1+\beta\cos\theta_0)/\sqrt{1-\beta^2}$ within the smooth front can be satisfied only for rather high densities of the plasma created behind a relativistic front or for incident angles close to $\theta_0^{\max}$ (Fig. 2). Moreover, it is known that for a smoothly inhomogeneous stationary warm plasma only about 50% of the incident wave energy can be transferred into Langmuir waves at the point of plasma resonance [11], which is less than the values in Fig. 5. In addition, the effective resonance transformation into Langmuir waves occurs only in a narrow interval of incident angles due to the existence of a barrier between the reflection point and the plasma resonant point [11]. For a smooth ionization front, the general pattern of transformation should be the same as for a stationary plasma: the plasma resonance will be damped by convective spatial dispersion rather than by thermal dispersion [12]. At the same time, the energy transferred to the free streaming mode increases for smooth ionization fronts [3].

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[1] V. I. Semenova, Radiophys. Quantum Electron. **10**, 599 (1967).
[2] V. I. Semenova, Radiophys. Quantum Electron. **15**, 505 (1972).
[3] M. Lampe, E. Ott, and J. H. Walker, Phys. Fluids **21**, 42 (1978).
[4] M. I. Bakunov and A. V. Maslov, IEEE Trans. Plasma Sci. **27**, 655 (1999).
[5] R. L. Savage, Jr., R. P. Brogle, W. B. Mori, and C. Joshi, IEEE Trans. Plasma Sci. **21**, 5 (1993).
[6] P. Muggli, R. Liou, C. H. Lai, J. Hoffman, T. C. Katsouleas, and C. Joshi, Phys. Plasmas **5**, 2112 (1998).
[7] W. B. Mori, Phys. Rev. A **44**, 5118 (1991).
[8] R. L. Savage, Jr., C. Joshi, and W. B. Mori, Phys. Rev. Lett. **68**, 946 (1992).
[9] J. M. Dias, C. Stenz, N. Lopes, X. Badiche, F. Blasco, A. Dos Santos, L. Oliveira e Silva, A. Mysyrowicz, A. Antonetti, and J. T. Mendonca, Phys. Rev. Lett. **78**, 4773 (1997).
[10] See, e.g., F. Krausz, T. Brabec, M. Schnürer, and C. Spielmann, Opt. Photonics News **9** (7), 46 (1998) and references therein.
[11] D. W. Forslund, J. M. Kindel, K. Lee, E. L. Lindman, and R. L. Morse, Phys. Rev. A **11**, 679 (1975).
[12] S. V. Bulanov, L. M. Kovrizhnykh, and A. S. Sakharov, Zh. Éksp. Teor. Fiz. **72**, 1809 (1977) [Sov. Phys. JETP **45**, 949 (1977)].